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CHARTS FOR RAPID ANALYSIS OF 45° STRAIN-ROSETTE DATA

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SUMMARY

Charts are presented for rapidly determining the principal strains and stresses, the maximum shear strain and stress, and the orientation of principal axes from data on 45° strain rosettes. The charts may be used for analyzing the conventional data consisting of strains measured along three gage lines 45° apart, but their application is more direct if the rosette data are obtained by means of special circuits that require the use of four gages 45° apart.

INTRODUCTION

The determination of the principal strains and stresses at the surface of a member under load is usually accomplished by means of strain rosettes. A strain rosette consists of three or more independent strain gages placed in close proximity and usually orientated at equal angles to one another. The rosette is attached to the surface of the test member at the point at which it is desired to determine the principal strains and stresses, and strain indications are observed for the directions along which the gages are orientated. The observed strains are then reduced to principal strains and stresses by a consideration of the laws of strain and stress distribution at the point under test.

The mathematical equations for reducing strain-rosette data are well established and are summarized in reference 1 (pp. 38-40). Hill (reference 2) has devised a semigraphical method for analyzing the data. Combination charts and nomographs for use with data from four gages 45° apart have been prepared by Stang and Greenspan. (See reference 3.) Klemperer (reference 4) has developed an electrical computer for automatic analysis of strain-rosette data, and in reference 5 Murray has suggested a mechanical computer for this purpose. Other methods of reducing strain-rosette data are described in the discussion of reference 5 and in references 6 to 11.

Although the ideal solution of the rosette problem is by the use of electrical or mechanical computers, such computers are not available at all times. A chart by means of which strain-rosette data can be rapidly reduced with little computation and without the use of drafting instruments is very useful for occasional and repeated calculations. The object of this report is to present charts for use with three-gage 45° rosettes. From a single chart the maximum shear strain, the major and minor principal strains, and the orientation of the principal axes may be determined. If stress quantities are desired, the maximum shear stress, the major and minor principal stresses, and the orientation of the principal axes may be obtained from a second chart. A chart (suggested by Meier in his discussion of reference 5) is included for converting principal strains, however obtained, into corresponding principal stresses. The charts involving stresses are strictly applicable only to steel of an elastic modulus of 30×10^6 pounds per square inch and a Poisson's ratio of 0.3. For application to structural materials of a different modulus of elasticity but of the same Poisson's ratio, the chart values may be rapidly converted by the aid of single proportionality factors. If great accuracy is desired, individual charts may be constructed that take into account the exact elastic modulus and Poisson's ratio for any desired material. The method of construction is presented in the appendix.

The charts contain several families of lines, which may prove objectionable in repeated calculations. "Mechanical charts," in which most of the lines are replaced by two sliding hairlines and one pivoted hairline, have been found to yield rapid and accurate results. Such charts are also described.

Although the charts are suitable for analyzing the usual strain data from a three-gage 45° rosette, their application is more direct if special circuits requiring the use of four gages 45° apart are used to obtain the rosette data. Diagrams of the basic circuits for measuring the three strain quantities that are fundamental in rosette analysis are presented herein.

If only maximum shear strains and stresses and orientation of principal axes are desired, two observations on a three-gage rosette are sufficient. No switches are required in the resistance-sensitive elements of the circuit, and the results are independent of temperature changes during the investigation.

The charts were constructed at the Aircraft Engine Research Laboratory of the NACA during the spring of 1943 for use in reducing data from rosettes attached to crankcases and other engine parts.

The following notation is used in the charts:

$\epsilon_1, \epsilon_2, \epsilon_3$ observed strain indications on gages 1, 2, and 3 of a 45° rosette, inches per inch. Gage 1 is the reference gage. Gage 2 is orientated at 45° and gage 3 is orientated at 90° positive counterclockwise to gage 1.

ϵ_p, ϵ_q major and minor principal strains at point of surface under test, inches per inch

γ_{max} maximum shear strain at point of surface under test, inches per inch

σ_p, σ_q major and minor principal stresses at point of surface under tests, pounds per square inch

τ_{max} maximum shear stress at point of surface under test, pounds per square inch

θ_p, θ_q angles of axes of major and minor principal stresses and strains. Degrees are measured positive counterclockwise from reference gage 1 of rosette.

E modulus of elasticity, pounds per square inch

μ Poisson's ratio

USE OF CHARTS

The charts presuppose a knowledge of the strain quantities $(\epsilon_2 - \epsilon_1)$, $(\epsilon_2 - \epsilon_3)$, and $(\epsilon_1 + \epsilon_3)$. These strain quantities may be calculated from the individual strain-gage readings of the rosette, or they may be observed directly by means of special circuits described in a later section of this report.

The three foregoing fundamental strain quantities are all multiplied by some power of 10 to bring them within the range of the scales in figures 1, 2, and 3. In very rare instances, it may be necessary to use a multiplying factor of 2 or 3 in order to bring the data to a convenient portion of the chart. In most cases, factors of 10^{-1}, 1, and 10 will suffice. The charts of figures 1, 2, or 3 are then employed for rapid analysis of the data in terms of either strain or stress. Detailed instructions on the method of their use are given directly on the charts. In order to obtain true values of strain and stress quantities, the values obtained

from the charts are divided by the same factor as that used to mul-
tiply the strain quantities $(\epsilon_2 - \epsilon_1)$, $(\epsilon_2 - \epsilon_3)$, and $(\epsilon_1 + \epsilon_3)$.
Orientation of principal axes are unaffected by the multiplying
factor; then, regardless of the multiplying factor, the chart indi-
cation yields the true orientation of the principal axes.

Figure 1 is used when it is desired to reduce rosette data to
terms of maximum and minimum strains at the point of surface under
test. This particular chart is universal in application and may be
applied to data on any material.

The chart of figure 2 (see the discussion by Meier of refer-
ence 5) has general application, within the elastic limit of the
material, in converting principal strains determined either from
figure 1 or by any other method into corresponding principal stresses
and maximum shear stress. The chart is direct reading for steel
that has a modulus of elasticity of 30×10^6 pounds per square inch
and a Poisson's ratio of 0.3. For other structural metals, Poisson's
ratio is sufficiently close to 0.3 that relatively small errors are
introduced by calculating the true stresses from chart values by
the relations

$$\left. \begin{array}{c} \sigma_p' = \dfrac{E'}{30 \times 10^6} \sigma_p \\[4mm] \sigma_q' = \dfrac{E'}{30 \times 10^6} \sigma_q \\[4mm] \tau_{max}' = \dfrac{E'}{30 \times 10^6} \tau_{max} \end{array} \right\} \qquad (1)$$

where

σ_p, σ_q, τ_{max} principal stresses and maximum shear stress obtained
                from chart (fig. 2), pounds per square inch

σ_p', σ_q', τ_{max}' true principal stresses and maximum shear stress for
                material under test, pounds per square inch

E' modulus of elasticity of material under test, pounds
                per square inch

When principal strains are not required and it is desired to
proceed immediately from rosette data to data on maximum and minimum
stress, figure 3 may be directly applied, provided that the strains
are within the elastic limit of the material. Equations (1) are

applied to the chart values of stress when the modulus of elasticity of the test material is not 30×10^6 pounds per square inch. The chart value of the orientation of the principal axes is unaffected by material.

Analysis of data by figures 1 and 2. - The use of figures 1 and 2 will be demonstrated by means of an illustrative example:

Strain quantity
(microin./in.)

$(\epsilon_2 - \epsilon_1)$ $(\epsilon_2 - \epsilon_3)$ $(\epsilon_1 + \epsilon_3)$

35   20   30

1. Multiply all strain quantities by 10. Data become:

$(\epsilon_2 - \epsilon_1)$ $(\epsilon_2 - \epsilon_3)$ $(\epsilon_1 + \epsilon_3)$

350   200   300

2. In figure 1, locate point with coordinates $(\epsilon_2 - \epsilon_1) = 350$ and $(\epsilon_2 - \epsilon_3) = 200$. At this point $\gamma_{max} = 570$ microinches per inch and $\theta_p = 52.6$. The scale marked $\oplus$ is used for determination of orientation of the axes because $(\epsilon_2 - \epsilon_3)$ is positive.

3. Follow a circle from the point located in 2 until the vertical axis is intersected at a value of 403 microinches per inch. Follow the horizontal direction from this point to an abscissa $(\epsilon_1 + \epsilon_3)$ of 300 microinches per inch; the major and minor principal strains are 435 and -135 microinches per inch, respectively.

4. From figure 2, locate point where $\epsilon_q = -135$ microinches per inch and $\epsilon_p = 435$ microinches per inch. The major and minor principal stresses are, respectively, 13,000 and -150 pounds per square inch. The maximum shear stress is 6600 pounds per square inch.

5. Divide all strains and stresses by the original multiplying factor 10; the final results are:

ϵ_p	ϵ_q	γ_{max}	σ_p	σ_q	τ_{max}	θ_p
(microin. per in.)			(lb per sq in.)			(deg counter-clockwise)
43.5	-13.5	57.0	1300	-15	660	52.6

Analysis of data by figure 3. - The use of figure 3, using the same example, is as follows:

1. Multiply strain quantities by 10 as before.

2. At the point with coordinates $(\epsilon_2 - \epsilon_1) = 350$ microinches per inch and $(\epsilon_2 - \epsilon_3) = 200$ microinches per inch, $\tau_{max} = 6600$ pounds per square inch and $\theta_p = 52.6°$ positive counterclockwise to direction of gage 1.

3. As in direction 3 of the preceding analysis, locate final point. From families of inclined lines major and minor principal stresses are 13,000 and -150 pounds per square inch, respectively.

4. Divide stresses by original multiplying factor 10, and the stresses are as given in the foregoing table.

Accuracy of analysis. - The foregoing stress and strain quantities, as determined from the charts, are within 1 percent of the computed values. In general, strain quantities may be obtained from the charts to within 5 microinches per inch and stress quantities to within 200 pounds per square inch. The accuracy of the final result depends on the multiplication factor used in the analysis. In the illustrative example, the final stresses and strains were thus obtained by multiplying the chart values by 0.1; hence, for a chart accuracy of 5 microinches per inch and 200 pounds per square inch, the final stresses and strains are accurate to within 0.5 microinch per inch and 20 pounds per square inch.

MECHANICAL CHARTS

It is possible to replace several of the families of lines in figures 1 and 3 by introducing two sliding hairlines and one pivoted hairline with an index rider. Such a mechanical chart, which corresponds to figure 1, is shown in figure 4.

The operation of the mechanical chart is deduced directly from the procedure followed in using the previously described charts. The vertical and horizontal hairlines simply replace the corresponding coordinate families; the pivoted hairline replaces the radial and the circular families of lines. The following procedure is required to operate the mechanical chart:

1. Move the vertical hairline to a coordinate of $(\epsilon_2 - \epsilon_1)$ and the horizontal hairline to a coordinate of $(\epsilon_2 - \epsilon_3)$. Use the absolute value of $(\epsilon_2 - \epsilon_3)$ and disregard its algebraic sign.

2. Rotate the pivoted hairline until it passes through the intersection of the vertical and horizontal hairlines. Move the index rider on the pivoted hairline to the point of intersection. Read θ_p from the radial-line scale. Use the scale marked $\oplus$ if $(\epsilon_2 - \epsilon_3)$ is positive and the scale marked $\ominus$ if $(\epsilon_2 - \epsilon_3)$ is negative.

3. Rotate the pivoted hairline until it is in a vertical position and slide the horizontal hairline up to the index rider of the pivoted hairline. Read maximum shear strain from intersection of horizontal hairline and γ_{max} scale.

4. Move vertical hairline to coordinate of $(\epsilon_1 + \epsilon_3)$ and read ϵ_p and ϵ_q at intersection of vertical and horizontal hairlines.

CIRCUITS FOR MEASURING SUMS AND DIFFERENCES OF STRAINS

It is observed from the expressions in the appendix involving stresses and strains that the strains ϵ_1, ϵ_2, and ϵ_3 occur only in the combinations $(\epsilon_2 - \epsilon_1)$, $(\epsilon_2 - \epsilon_3)$, and $(\epsilon_1 + \epsilon_3)$; hence, if the circuits are arranged to be direct reading of these strain quantities, the amount of time required for analyzing rosette data will be reduced. Each observed quantity, furthermore, will be subject to only one observational error whereas, if the sum and difference quantities are obtained by calculation from the individual strain quantities, the results are subject to two observational errors. The problem is, then, to devise circuits for directly obtaining the three strain quantities $(\epsilon_2 - \epsilon_1)$, $(\epsilon_2 - \epsilon_3)$, and $(\epsilon_1 + \epsilon_3)$.

Basis of Circuits

Figure 5 shows a standard bridge circuit. If the bridge is initially balanced

$$R_a \times R_d = R_b \times R_c \tag{2}$$

Let R_a, R_b, R_c, and R_d, respectively, assume the increments ΔR_a, ΔR_b, ΔR_c, and ΔR_d such that the balance of the bridge is not destroyed. Then

$$(R_a + \Delta R_a) \times (R_d + \Delta R_d) = (R_b + \Delta R_b) \times (R_c + \Delta R_c)$$

or

$$R_a R_d + \Delta R_a R_d + \Delta R_d R_a + \Delta R_a \Delta R_d = R_b R_c + \Delta R_b R_c + \Delta R_c R_b + \Delta R_b \Delta R_c \tag{3}$$

If all increments of resistance are so small that the products of two increments may be neglected in equation (3) and, in addition, if all resistances are initially approximately equal, equation (3) reduces to

$$\Delta R_a + \Delta R_d = \Delta R_b + \Delta R_c$$

or

$$\Delta R_d = \Delta R_b + \Delta R_c - \Delta R_a \tag{4}$$

When the increments are arbitrary and the final balance does not necessarily result, the voltage across the galvanometer Δv is approximately proportional to

$$\Delta v \propto \Delta R_b + \Delta R_c - \Delta R_d - \Delta R_a \tag{5}$$

A basis is now provided for measurement of sums and differences of strains. Let R_a and R_b be gages 1 and 2 of a rosette, let R_c be a fixed resistor $(\Delta R_c = 0)$, and let R_d be a strain gage on a calibrated cantilever suitable for producing bridge balance subsequent to strain in R_b and R_c. Then, by equation (4), the increment ΔR_d required to produce the balance is numerically equal to the difference in resistance change of gage 2 and gage 1, which, in turn, is proportional to $(\epsilon_2 - \epsilon_1)$. By analogous arrangement, $(\epsilon_2 - \epsilon_3)$ may be obtained. If a deflection galvanometer or automatic voltage-recording equipment is used for measurement of strain, R_d may be a fixed resistor, and the galvanometer reading would yield a direct measure of the required strain differences.

In order to obtain a measure of $(\epsilon_1 + \epsilon_3)$, R_a is made a fixed resistor, and R_b and R_c are gages 1 and 3, respectively. An alternate method of obtaining a measure of $(\epsilon_1 + \epsilon_3)$ is to introduce a fourth gage at right angles to gage 2. This gage is known as gage 4 of the rosette. Inasmuch as it can be deduced from equation (1.33) of reference 1 that

$$\epsilon_1 + \epsilon_3 = \epsilon_2 + \epsilon_4 \tag{6}$$

gages 2 and 4 may be made the opposite arms of the bridge and the sum of their strains taken as a measure of $(\epsilon_1 + \epsilon_3)$. This introduction of a fourth gage simplifies the instrumentation, but it does not serve the purpose, as a fourth gage usually does, of providing a set of redundant data to check the readings of the other gages.

Circuit for Four–Gage Rosette

The diagram of the basic circuit for obtaining the three quantities $(\epsilon_2 - \epsilon_1)$, $(\epsilon_2 - \epsilon_3)$, and $(\epsilon_1 + \epsilon_3)$ is shown in figure 6. The balancing resistors – R_1, R_2, R_3 – serve to bring the galvanometer to a null deflection for switch positions 1, 2, and 3, respectively, prior to application of strain on the gages. Subsequent to application of strain, the increment in the balancing standard R_2 necessary to bring the galvanometer to a null reading, when the switch is in position 1, is proportional to $(\epsilon_2 - \epsilon_1)$. In position 2 of the galvanometer switch the increment in R_2 required for balance is proportional to $(\epsilon_2 - \epsilon_3)$. In position 3 the increment is proportional to $(\epsilon_2 + \epsilon_4)$, which, in turn, is equal to $(\epsilon_1 + \epsilon_3)$.

Circuit for Successive Readings on Many Rosettes of Four Gages

When it is desired to obtain readings from many rosettes, the circuit of figure 7 may be used. The arrangement requires a doublepole, multithrow switch, in which the number of throws is equal to the number of rosettes to be tested. A single–pole, triple–throw switch is required for each rosette to be tested. The galvanometer may be either null type or deflection type, or it may be replaced by automatic voltage–unbalance–recording equipment. For use with a null–type galvanometer the resistors R must be variable by known amounts in order to bring the bridge to balance for each of the three positions of the position selector. With the rosette selector set to a given rosette the change in resistance of R required to produce bridge balance for positions 1, 2, and 3 of the position selector yields $(\epsilon_2 - \epsilon_1)$, $(\epsilon_2 - \epsilon_3)$, and $(\epsilon_2 + \epsilon_4 = \epsilon_1 + \epsilon_3)$,

respectively. For use with a deflection galvanometer or automatic
voltage-recording equipment the resistors R must be fixed, and
the unbalance of the galvanometer for each of the selector positions
yields the foregoing strain quantities.

The circuit of figure 7 can, of course, be used for the general
purpose of successively measuring the indivudual strains on many
gages. It is only necessary to insert fixed resistors between the
terminals allocated to gage 2 of the rosettes when the rosettes are
tested. Due consideration should also be given to the signs of the
strain indications. In positions 1 and 2 of the selector switch the
strains indicated are the negative of the true strain values.

Circuit for Measurement of Dynamic Strains

The circuit of figure 6 may be used for measurement of dynamic
stress by means of rosettes. The galvanometer is replaced by three
oscillographs with suitable preamplifiers, and the three basic
strain quantities $(\epsilon_2 - \epsilon_1)$, $(\epsilon_2 - \epsilon_3)$, and $(\epsilon_2 + \epsilon_4 = \epsilon_1 + \epsilon_3)$
are simultaneously obtained. For measurement of total stress and
audiofrequency alternating-current excitation voltage is used instead
of the battery shown in the figure. Suitable condensers are also
placed in parallel with the balancing resistors in order to effect
phase as well as resistance balance. The common terminal A of
all the inputs to the amplifiers may be grounded if necessary.

Circuit for Shear Values and Axes Orientation Only

Under some conditions it may be desirable to determine only
maximum shear strain, maximum shear stress, and orientation of the
principal axes. For this purpose the special circuits are ideally
suited. Only $(\epsilon_2 - \epsilon_1)$ and $(\epsilon_2 - \epsilon_3)$ need be measured; thus,
two measurements suffice where three would be required if ϵ_1, ϵ_2,
and ϵ_3 were individually determined. The two measurements of
$(\epsilon_2 - \epsilon_1)$ and $(\epsilon_2 - \epsilon_3)$ may be obtained with a standard rosette
of three gages. The basis of the circuits for this purpose is the
circuit of figure 6 with G_4, R_4, and position 3 of the selector
switch removed. Only positions 1 and 2 of the selector switch are
required. The modifications to the other circuit diagram (fig. 7)
for successive measurements on many rosettes are similar.

Temperature Compensation in Circuits

It is interesting to note that, if all gages have the same temperature coefficient of resistance, the readings $(\epsilon_2 - \epsilon_1)$ and $(\epsilon_2 - \epsilon_3)$ are inherently temperature-compensated; hence, determinations of maximum shear strain and stress are not affected by variation in ambient temperature. This fact is so because temperature variation, which manifests itself as equal strains in all directions at a point on a test specimen, does not introduce any shear strains nor does it alter the direction of the principal axes. If only shear values and orientation of principal axes are of interest, no regard need be given to temperature compensation.

The reading $(\epsilon_2 + \epsilon_4)$ is not inherently temperature-compensated. Effects of temperature variation may be minimized, however, by making R_2 and R_4 dummy gages mounted on a metal strip of the same material as that under test and placed in proximity to the test specimen. This procedure is possible only in the case when strains are observed with a deflection galvanometer or with automatic voltage-unbalance-recording equipment. Initial bridge balance in such a case is obtained by means of a large variable resistor that may be switched in parallel with either R_4 or G_4. Error due to the balancing resistor may be minimized by choosing all gage resistances very closely equal.

Unless many rosettes are to be analyzed and considerable time can be saved by obtaining $(\epsilon_2 + \epsilon_4)$ directly, the special circuits may not be advantageous for a determination of principal stresses. Instead, it may be better to use the temperature-compensated values of ϵ_1, ϵ_2, and ϵ_3 individually determined and, subsequently, to combine them into sums and differences for use with figures 1, 2, and 3. The special circuits are, however, particularly suited to the determination of maximum shear values and orientation of principal axes.

CONCLUDING REMARKS

The charts presented in this report are suitable for a rapid analysis of data from rosettes consisting of three gages 45° apart. Special circuits have been described that yield the rosette data in a form more suitable for analysis by the application of the charts. For a determination of maximum shear strain and orientation of principal axes, the special circuits are very satisfactory since they eliminate temperature-compensation problems and permit a complete

determination from only two observations of strain. The special
circuits for obtaining the data required in the determination of
principal strains and stresses, as well as shear values, require
four strain gages and present a problem of temperature compensation
(although no more serious than the usual problem of temperature com-
pensation in ordinary strain measurements). Unless a large number
of rosettes are to be analyzed, the special circuits may not afford
a saving in time over the usual strain-measuring circuits. They
are interesting in any case, however, in that they indicate how the
fundamental strain quantities can be obtained.

Aircraft-Engine Research Laboratory,
     National Advisory Committee for Aeronautics,
          Cleveland, Ohio, December 27, 1943.

APPENDIX

CONSTRUCTION OF CHARTS

Explanation of the construction of the various families of lines in the charts of figures 1, 2, and 3 is presented in order to indicate how charts similar to those of figures 2 and 3 may be constructed to be direct reading for any material. If numerous tests are to be performed on a given material, it is advisable to determine accurately the modulus of elasticity and the Poisson's ratio for the material and to construct a chart for the material similar to that of figure 3. A convenient method of effecting the determination is to apply several rosettes to a strip of the material and subsequently to stress the strip unilaterally. With the aid of figure 1 the rosette data may be analyzed to obtain the principal strains. The absolute value of the ratio of the minor principal strain to the major principal strain is Poisson's ratio, and the major principal strain, in conjunction with the known loads, may be used to determine the modulus of elasticity.

Maximum Shear Strain and Stress

The expression for maximum shear strain, by deduction from equations (1.30), (1.46), and (1.47) of reference 1, is

$$\gamma_{max} = \epsilon_p - \epsilon_q = \sqrt{2} \ \sqrt{(\epsilon_2 - \epsilon_1)^2 + (\epsilon_2 - \epsilon_3)^2} \qquad (7)$$

The coordinate axes of figure 1 are $(\epsilon_2 - \epsilon_1)$ and $(\epsilon_2 - \epsilon_3)$. It is clear that the distance from the origin to any point with coordinates $(\epsilon_2 - \epsilon_1)$ and $(\epsilon_2 - \epsilon_3)$ will be given by the square-root quantity containing strain differences in equation (7); hence, circles with centers at the origin are loci of constant maximum shear strain. The multiplying factor $\sqrt{2}$ may be taken into account by proper calibration of the concentric circles.

The expression for maximum shear stress is deduced from equations (1.43), (1.46), and (1.47) of reference 1.

$$\tau_{max} = \frac{\sqrt{2}}{2} \ \frac{E}{1 + \mu} \ \sqrt{(\epsilon_2 - \epsilon_1)^2 + (\epsilon_2 - \epsilon_3)^2} \qquad (8)$$

Again, concentric circles on a chart of coordinate axes $(\epsilon_2 - \epsilon_1)$ and $(\epsilon_2 - \epsilon_3)$ represent loci of constant τ_{max}. Such circles are

shown in figure 3 for steel of a modulus of elasticity of 30×10^6 pounds per square inch and a Poisson's ratio of 0.3. Construction of a chart for a different material of known modulus of elasticity and Poisson's ratio involves substitution of numerical values of E and μ in equation (8). Assumed values of τ_{max} are substituted in the equation, and the square-root quantity is calculated for each assumed value. The numerical value of the square-root quantity is the radius of the circle corresponding to its assumed τ_{max}.

Orientation of Major Principal Axis

The angle of the major principal axis measured positive counterclockwise from the direction of gage 1 is determined from equation (1.48) of reference 1 to be

$$\tan 2\theta_p = \frac{2\epsilon_2 - (\epsilon_1 + \epsilon_3)}{\epsilon_1 - \epsilon_3}$$

This equation may be rewritten

$$\tan 2\theta_p = \frac{(\epsilon_2 - \epsilon_3) + (\epsilon_2 - \epsilon_1)}{(\epsilon_2 - \epsilon_3) - (\epsilon_2 - \epsilon_1)}$$

from which may be deduced

$$\frac{\tan 2\theta_p + 1}{\tan 2\theta_p - 1} = \frac{\epsilon_2 - \epsilon_3}{\epsilon_2 - \epsilon_1} \tag{9}$$

If, in equation (9), a value of θ_p is assumed, the expression takes the form $(\epsilon_2 - \epsilon_3) = K (\epsilon_2 - \epsilon_1)$ where K is a constant depending on the assumed value of θ_p. In a plane of coordinate system $(\epsilon_2 - \epsilon_1)$ and $(\epsilon_2 - \epsilon_3)$, this equation represents a straight line through the origin. The locus of points of constant angle is thus a straight line through the origin.

In figures 1 and 3 the radial straight lines constitute the family from which the orientation of the principal axes may be determined. If the chart were complete, with four quadrants shown in order to provide for both positive and negative values of $(\epsilon_2 - \epsilon_1)$ and $(\epsilon_2 - \epsilon_3)$, a single scale would suffice to completely establish the angle of the major principal axes. In figures 1 and 3 only two quadrants are shown, which make two scales necessary.

Major Principal Strain and Stress

The expression for the major principal strain (equation (1.46), reference 1) is

$$\epsilon_p = \frac{(\epsilon_1 + \epsilon_3)}{2} + \frac{\sqrt{2}}{2}\sqrt{(\epsilon_2 - \epsilon_1)^2 + (\epsilon_2 - \epsilon_3)^2} \qquad (10)$$

In the determination of the maximum shear strain and the orientation of principal axes, the point with coordinates $(\epsilon_2 - \epsilon_1)$ and $(\epsilon_2 - \epsilon_3)$ has already been located. If the circle on which this point lies is followed until the vertical axis is intersected, a value of $(\epsilon_2 - \epsilon_3)'$ is obtained such that

$$(\epsilon_2 - \epsilon_3)' = \sqrt{(\epsilon_2 - \epsilon_1)^2 + (\epsilon_2 - \epsilon_3)^2} \qquad (11)$$

Equation (10) becomes

$$\epsilon_p = \frac{1}{2}(\epsilon_1 + \epsilon_3) + \frac{\sqrt{2}}{2}(\epsilon_2 - \epsilon_3)' \qquad (12)$$

If, now, a set of new axes for coordinates $(\epsilon_2 - \epsilon_3)'$ and $(\epsilon_1 + \epsilon_3)$ are imagined to be superimposed on the original set of axes, equation (12) represents a linear relation in $(\epsilon_1 + \epsilon_3)$ and $(\epsilon_2 - \epsilon_3)'$. When consecutive values of ϵ_p are assumed, equation (12) yields, for each assumed value of ϵ_p, a straight-line variation of $(\epsilon_1 + \epsilon_3)$ with $(\epsilon_2 - \epsilon_3)'$.

In figure 1 such a family of straight lines is shown. From the foregoing discussion it is evident that the procedure for determining the major principal strain is as follows: Locate a point the coordinates of which are $(\epsilon_2 - \epsilon_1)$ and $(\epsilon_2 - \epsilon_3)$; follow the circle on which this point lies until the vertical axis is intersected; a value of $(\epsilon_2 - \epsilon_3)' = \sqrt{(\epsilon_2 - \epsilon_1)^2 + (\epsilon_2 - \epsilon_3)^2}$ is obtained; from the point of intersection, follow the horizontal direction to an abscissa of $(\epsilon_1 + \epsilon_3)$. The line of the family of ϵ_p that passes through the final point located defines a value of ϵ_p that satisfies equation (12) and is therefore the major principal strain.

The equation for major principal stress by deduction from equations (1.53), (1.46), and (1.47) of reference 1 is

$$\sigma_p = \frac{E}{1-\mu^2}\left(\epsilon_p + \mu\,\epsilon_q\right) = E\left[\frac{\epsilon_1 + \epsilon_3}{2(1-\mu)} + \frac{\sqrt{2}}{2(1+\mu)}\sqrt{(\epsilon_2 - \epsilon_1)^2 + (\epsilon_2 - \epsilon_3)^2}\right] \quad (13)$$

In a manner analogous to that used in obtaining equation (12) from equation (10), equation (13) is reduced to

$$\sigma_p = E\left[\frac{\epsilon_1 + \epsilon_3}{2(1-\mu)} + \frac{\sqrt{2}}{2(1+\mu)}(\epsilon_2 - \epsilon_3)'\right] \quad (14)$$

The relation is again linear, and the lines representing a constant value of σ_p are obtained by substituting this value of σ_p in equation (14); the equation of the straight line between $(\epsilon_1 + \epsilon_3)$ and $(\epsilon_2 - \epsilon_3)'$ is thereby obtained.

In figure 3 a family of lines for σ_p are drawn for steel of a modulus of elasticity of 30×10^6 pounds per square inch and a Poisson's ratio of 0.3. In order to obtain charts for other materials, the procedure is to substitute the proper values of E and μ in equation (14) and thus to obtain the equation of a straight line between $(\epsilon_1 + \epsilon_3)$ and $(\epsilon_2 - \epsilon_3)'$ for any constant value of σ_p. When values of σ_p at equal intervals are assumed, a family of equidistant parallel lines results. Each line is labeled to correspond to the value of σ_p defining it.

Minor Principal Strain and Stress

The analysis of the families of lines defining the minor principal strains and stresses is similar to the foregoing analysis for the major principal strains and stresses. The only difference is that a negative sign precedes the square-root quantity, which affects the slopes of the families of lines.

Principal Stresses from Principal Strains

The relations between principal stresses and principal strains, deduced from equations (1.39) of reference 1, are

$$\sigma_p' = \frac{E}{1 - \mu^2}\left[\epsilon_p + \mu\,\epsilon_q\right] \tag{15}$$

and

$$\sigma_q = \frac{E}{1 - \mu^2}\left[\epsilon_q + \mu\,\epsilon_p\right] \tag{16}$$

If values are assigned to σ_p or σ_q, straight-line relations in ϵ_p and ϵ_q result. This fact suggests a chart such as figure 2 for immediate conversion of principal strains into principal stresses. Figure 2 is applicable to steel of an elastic modulus of 30×10^6 pounds per square inch and a Poisson's ratio of 0.3. In order to use the chart for a structural material with a Poisson's ratio of approximately 0.3 and an elastic modulus E', equation (1) may be applied to values of ϵ_p and ϵ_q obtained from figure 2.

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1. Locate point with horizontal coordinate $(\epsilon_2 - \epsilon_1)$ and vertical coordinate $(\epsilon_2 - \epsilon_3)$. Use absolute value of $(\epsilon_2 - \epsilon_3)$, disregarding its sign.
2. At this point read maximum shear strain from circular-arc scale and orientation of major principal axis from radial-line scale. If $(\epsilon_2 - \epsilon_3)$ is positive, use scale designated ⊕; if $(\epsilon_2 - \epsilon_3)$ is negative, use scale designated ⊖.

3. From point located follow circular arc until vertical axis is intersected. From point of intersection follow horizontal line to an abscissa of $(\epsilon_1 + \epsilon_3)$.
4. From families of inclined lines read major and minor principal strains.
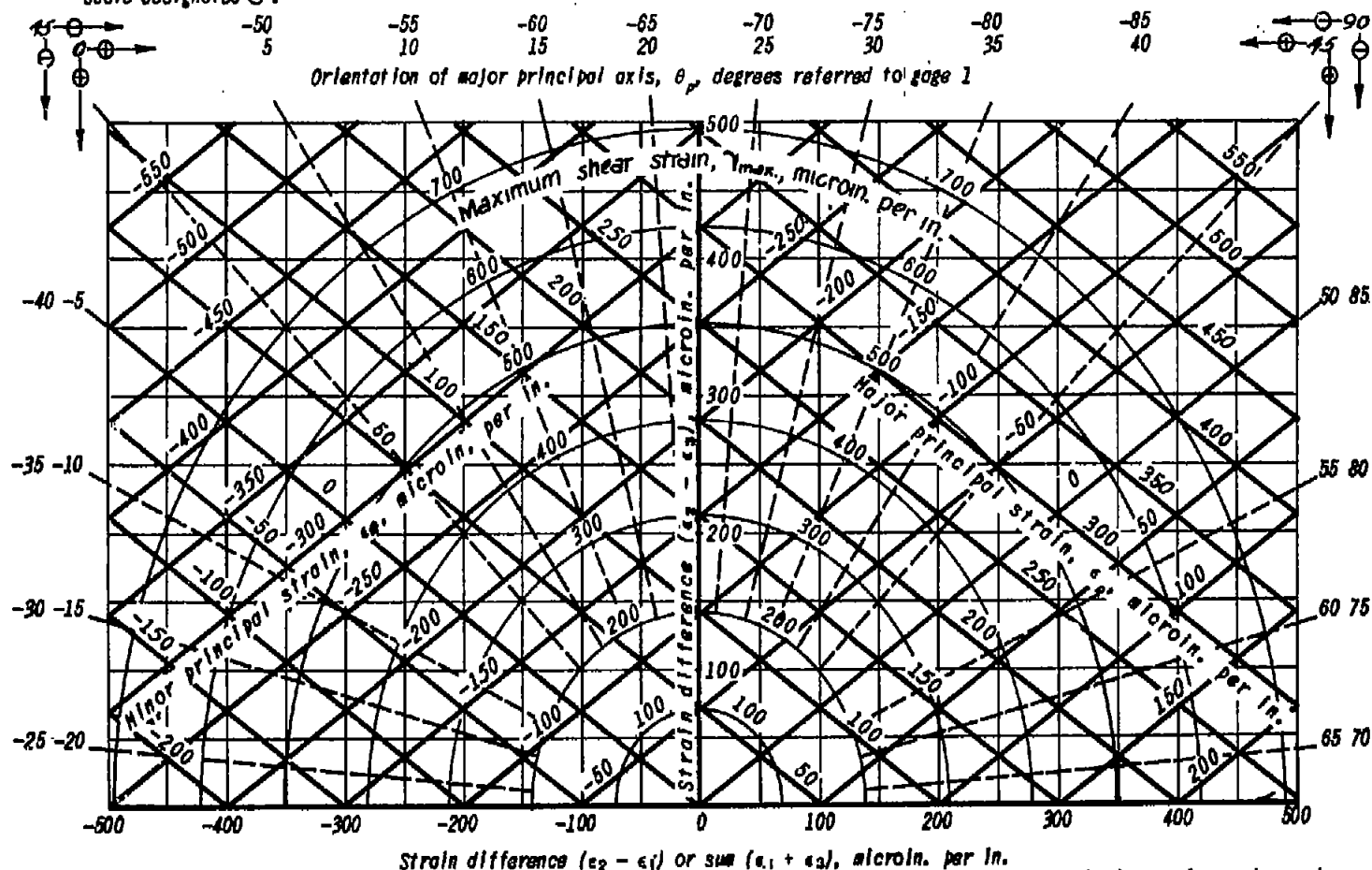
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Figure 1. – Chart for determining principal strains, maximum shear strain, and orientation of principal axes from observed strains from a three-or four-gage 45° strain rosette.

Fig. 1

1. Locate point with horizontal coordinate $(\epsilon_2 - \epsilon_1)$ and vertical coordinate $(\epsilon_2 - \epsilon_3)$. Use absolute value of $(\epsilon_2 - \epsilon_3)$ disregarding its sign.

2. At this point read maximum shear stress from circular-arc scale, and orientation of major principal axis from radial-line scale. If $(\epsilon_2 - \epsilon_3)$ is positive, use scale designated $\oplus$; if $(\epsilon_2 - \epsilon_3)$ is negative, use scale designated $\ominus$.

3. From point located follow circular arc until vertical axis is intersected. From point of intersection follow horizontal line to an abscissa of $(\epsilon_1 + \epsilon_3)$.

4. From families of inclined lines read major and minor principal stresses.
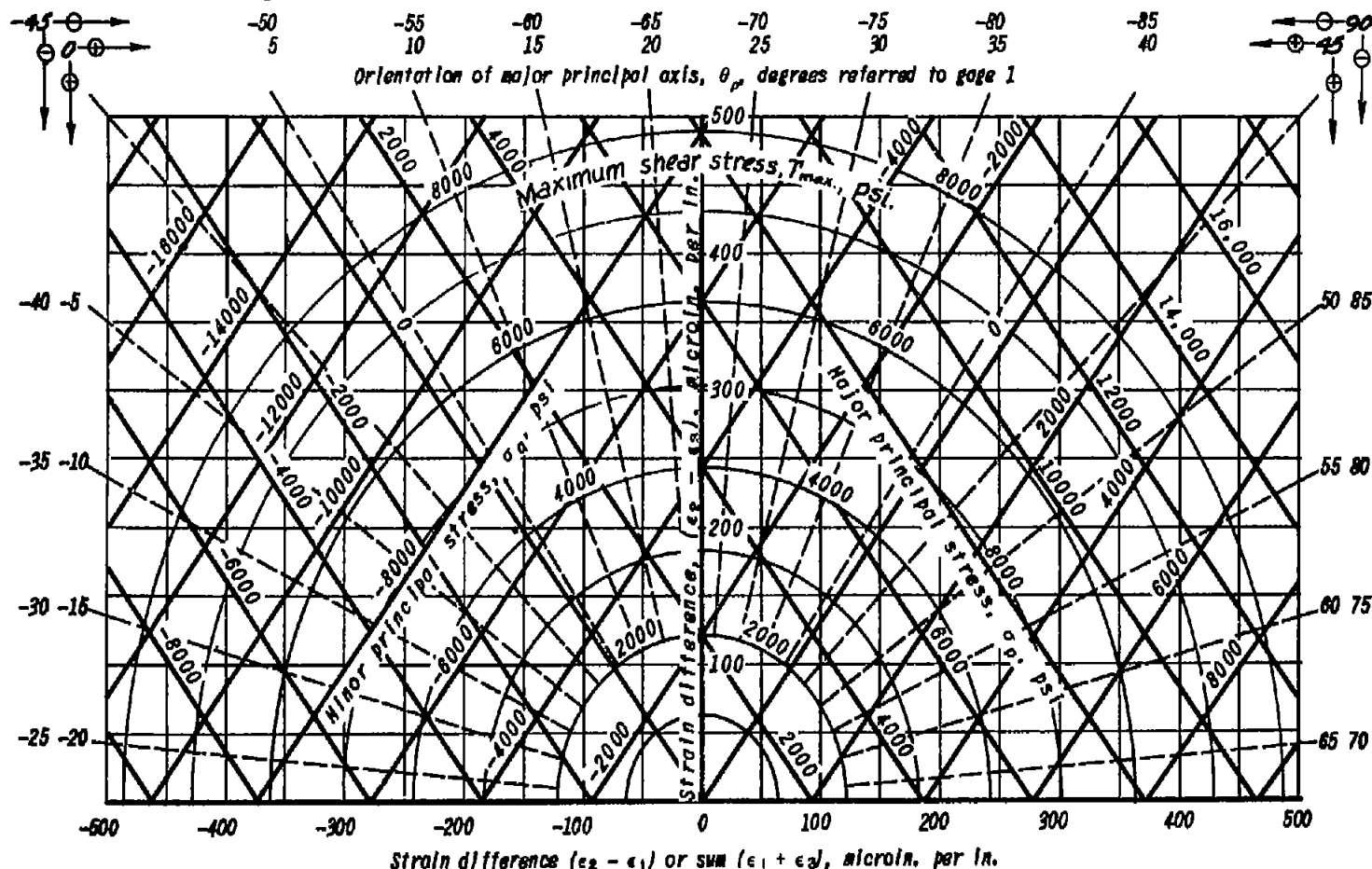
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Orientation of major principal axis, θ_p, degrees referred to gage 1

Maximum shear stress, τ_{max}, psi.

Major principal stress, σ_1, psi.

Minor principal stress, σ_2, psi.

Strain difference, $(\epsilon_2 - \epsilon_3)$, microin. per in.

Strain difference $(\epsilon_2 - \epsilon_1)$ or sum $(\epsilon_1 + \epsilon_3)$, microin. per in.

Figure 3. – Chart for determining principal stresses, maximum shear stress, and orientation of principal axes from observed strains from a three-or four-gage 45° strain rosette. Steel: modulus of elasticity, 30×10^6 pounds per square inch; Poisson's ratio, 0.3.
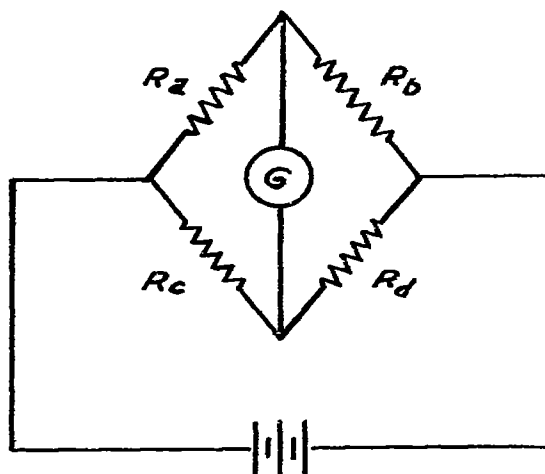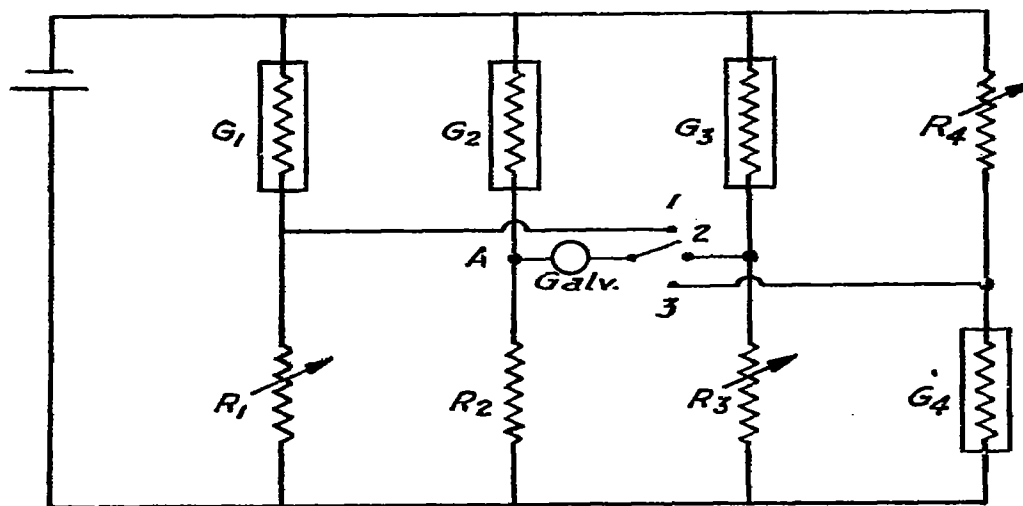
Figure 5.- Standard bridge circuit.

Figure 6.- Diagram of basic circuit for obtaining the
three strain quantities fundamental in analysis of data
from a 4-gage 45° strain rosette. G_1, G_2, G_3 and G_4 gages of
rosette ; R_1, R_3, R_4 balancing resistors for null deflection
of galvanometer in positions 1,2,3 ; R_2 standard for re-
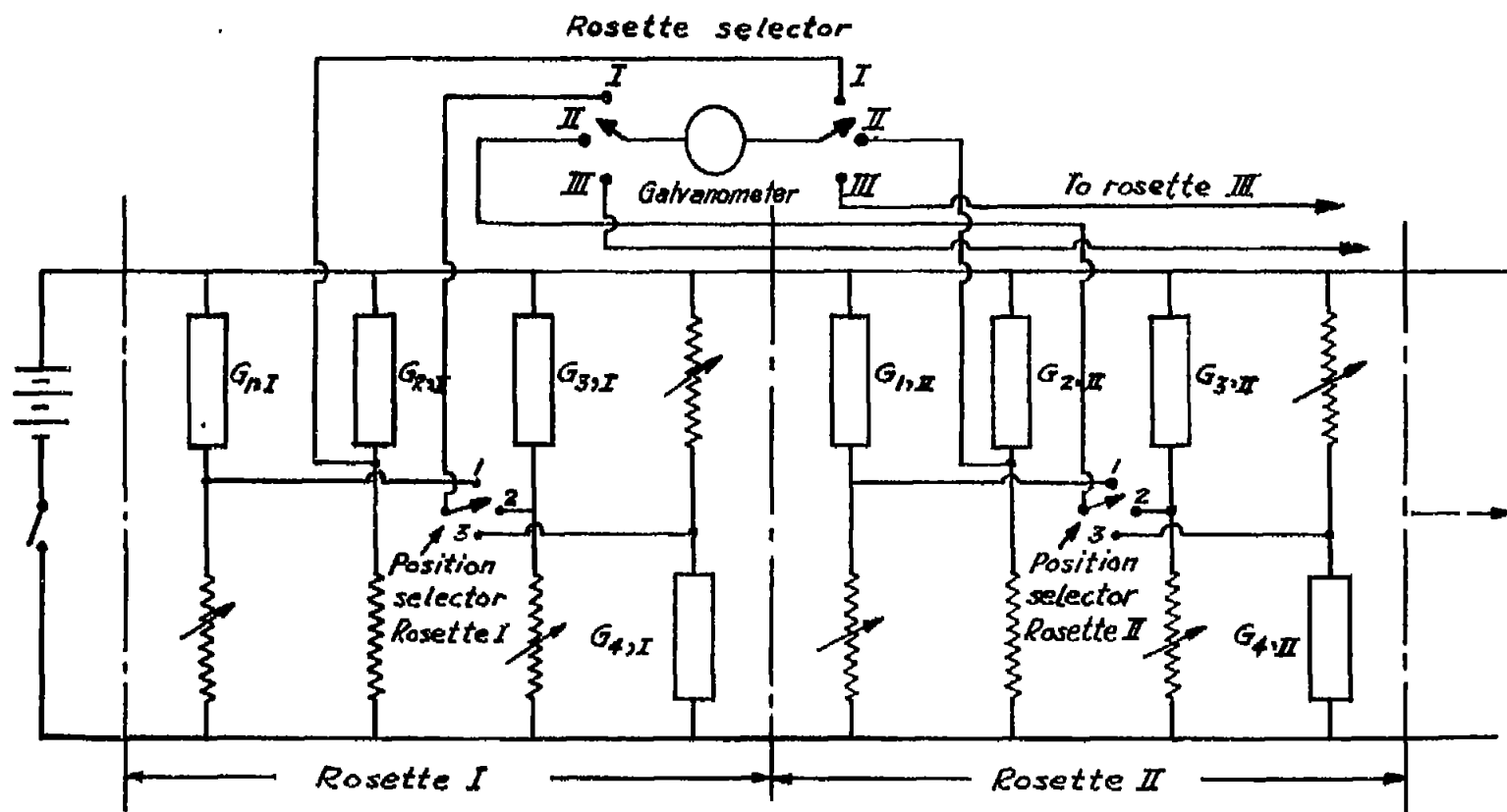balancing after gages have been strained.

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Figure 7.- Circuit diagram for obtaining the three fundamental strain quantities from many 4-gage 45° rosettes successively.

Fig. 7